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THE PROPERTIES OF CLUSTERS OF IMPURITY IN A TURBULENT MEDIUM†

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Using Kraichnan's model the specific properties of localization of an inertial and floating impurity in a turbulent medium are studied for various relations between the divergent and vortical part of the velocity field of impurity particles. © 2004 Elsevier Ltd. All rights reserved.

In recent years the properties of intermittency and stochastic localization of impurity in randomly moving media have been intensively studied both theoretically and experimentally (see, for example, [1–6]). It has already been established (see, for example, [1, 5]) that the principal mechanism of localization is related to the presence of a divergent component of the velocity field of the medium, which arises when there is inertial motion of the particles [7]. Correspondingly, it is assumed below that the impurity velocity field possesses both a vortical and divergent component; the field itself is described by Kraichnan's model [8] with a specified correlation tensor.

An analysis of the properties of impurity localization needs appropriate methods of statistical description. One such method, which is based on an analysis of the mean density around an arbitrarily chosen particle (henceforth we shall call this the tagged particle) and the conditional distribution of relative diffusion of particles, is proposed below. On the basis of the proposed analysis it becomes possible to calculate the means mass and specific sizes of clusters and to trace how the form of the clusters changes depending on the ratio of the divergent and vortical components of the particle velocities.

1. THE MEAN DENSITY AROUND THE TAGGED PARTICLE

The most effective method of providing a statistical description of the intermittency and stochastic localization of an impurity in a turbulent medium is to investigate the Lagrangian statistical characteristics of the impurity (see, for example, [9–12]). The mean density around the tagged particle, which will be be introduced below, is one of a variety of Lagrangian characteristics of the impurity.

Consider the density of an impurity particle of unit mass

$$\tilde{n}(\mathbf{z}, t; \boldsymbol{\xi}, \mathbf{s}) = \delta(\mathbf{X}(\boldsymbol{\xi}+\mathbf{s}, t) - \mathbf{X}(\boldsymbol{\xi}, t) - \mathbf{z}) \tag{1.1}$$

Here $\mathbf{X}(\boldsymbol{\xi}, t)$ are the actual coordinates of the tagged particle, $\mathbf{s}$ are the coordinates of the remaining particles in a Lagrangian system of coordinates with centre at the point $\boldsymbol{\xi}$ where the tagged particle is located, and $\mathbf{z}$ are the Eulerian coordinates, measured from the tagged particle situated at the point $\mathbf{X}(\boldsymbol{\xi}, t)$ at the current instant t. In this paper we take the particle coordinates at the initial instant $t = 0$ as the Lagrangian coordinates. In particular

$$\mathbf{X}(\boldsymbol{\xi}, 0) = \boldsymbol{\xi}, \quad \mathbf{X}(\boldsymbol{\xi}+\mathbf{s}, 0) = \boldsymbol{\xi}+\mathbf{s}$$

Suppose the initial density around the tagged particle $n_0(\mathbf{s})$ is known. Then, from relation (1.1) we have the following mean density around the tagged particle

$$\langle n_c(\mathbf{z}, t; \boldsymbol{\xi})\rangle = \int g(\mathbf{z}; \boldsymbol{\xi}, \mathbf{s}, t) n_0(\mathbf{s}) d\mathbf{s} \tag{1.2}$$

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This relation involves the distribution of the vector joining two particles

$$g(\mathbf{z};\, \boldsymbol{\xi}, \mathbf{s}, t) = \langle \delta(\mathbf{X}(\boldsymbol{\xi}+\mathbf{s}, t) - \mathbf{X}(\boldsymbol{\xi}, t) - \mathbf{z}) \rangle, \quad g(\mathbf{z};\, \boldsymbol{\xi}, \mathbf{s}, t=0) = \delta(\mathbf{s}-\mathbf{z})$$

If the random field of the impurity velocity $\boldsymbol{v}(\mathbf{x}, t)$ is statistically uniform, the dependence on $\boldsymbol{\xi}$ in equality (1.2) vanishes and this equality takes the form

$$\langle n_c(\mathbf{z}, t) \rangle = \int g(\mathbf{z};\, \mathbf{s}, t) n_0(\mathbf{s}) d\mathbf{s} \tag{1.3}$$

Suppose the equation for the distribution of the relative diffusion $g(\mathbf{z}, \mathbf{s}, t)$ is known

$$\partial g/\partial t = \mathscr{L} g, \quad g(\mathbf{z};\, \mathbf{s}, t=0) = \delta(\mathbf{z}-\mathbf{s}) \tag{1.4}$$

where $\mathscr{L}$ is some operator in the space $\mathbf{z}$. Then the mean density around the tagged particle conforms to Cauchy's problem

$$\partial \langle n_c \rangle / \partial t = \mathscr{L} \langle n_c \rangle, \quad \langle n_c(\mathbf{z}, t=0) \rangle = n_0(\mathbf{z}) \tag{1.5}$$

The mean density around the tagged particle enables one to judge the masses and characteristic sizes of the cluster. We shall show this using the example of the steady density of the cluster around the tagged particle

$$n_{\mathrm{st}}(\mathbf{z}) = \lim_{t \to \infty} \langle n_c(\mathbf{z}, t) \rangle \tag{1.6}$$

If the mean impurity density is uniform, $\langle n(\mathbf{x}, t) \rangle = n_0$, the above-mentioned steady density around the tagged particle conforms to the boundary-value problem

$$\mathscr{L} n_{\mathrm{st}}(\mathbf{z}) = 0, \quad n_{\mathrm{st}}(\mathbf{z})\big|_{|\mathbf{z}| \to \infty} = n_0 \tag{1.7}$$

The condition at infinity takes into account the fact that the remaining clusters make up a "background" with a mean density n_0. An excess of the cluster density over the mean density is specified by the function

$$\mathscr{F}(\mathbf{z}) = n_{\mathrm{st}}(\mathbf{z}) - n_0$$

We will define the mean mass of the cluster by the function

$$M = \int \mathscr{F}(\mathbf{z}) d\mathbf{z} \tag{1.8}$$

The function $\mathscr{G}(\mathbf{z})$, which specifies the particle distribution around the centre of the cluster, will be called the mean profile of the cluster. The mean profile is not identical with $\mathscr{F}(\mathbf{z})$, since the tagged particle may be located at any point of the cluster. This fact is expressed by the equality

$$\mathscr{F}(\mathbf{z}) = \langle \mathscr{G}(\mathbf{z} - \mathbf{z}_{\mathrm{tag}}) \rangle \tag{1.9}$$

where $\mathbf{z}_{\mathrm{tag}}$ is the coordinate of tagged particle in the system of coordinates with origin at the cluster centre. It is natural to take a distribution of coordinates of the tagged particle, over which the averaging is carried out in equality (1.9), in the form

$$g(\mathbf{z}) = \mathscr{G}(\mathbf{z})/M$$

Expanding the mean in (1.9) using this distribution, we arrive at an integral equation in the mean profile of the cluster

$$\mathscr{G}(\mathbf{z}) \otimes \mathscr{G}(\mathbf{z}) = M \mathscr{F}(\mathbf{z})$$

In the case of an isotropic medium let us find a steady solution of Eq. (1.5) using Kraichnan's model [10]. We recall that in Kraichnan's model it is assumed that the velocity field of the turbulent medium is δ-correlated in time, and the correlation tensor of the velocity field $\boldsymbol{v}(\mathbf{x}, t)$ of the medium is assumed to be specified by the relation

$$\langle v_i(\mathbf{x}, t) v_j(\mathbf{x}+\mathbf{z}, t+\tau) \rangle = \mathscr{D}_{ij}(\mathbf{z}) \delta(\tau)$$

This formulae includes the diffusivity tensor of the field of particle velocities

$$\mathcal{D}_{ij}(\mathbf{z}) = -\delta_{ij}\Delta A_e(z) + \frac{\partial^2}{\partial z_i \partial z_j}[A_e(z) - A_p(z)]$$

expressed in terms of scalar fields responsible for the divergent part of the velocity (A_p) and its vortical part (A_e).

Within the framework of Kraichnan's model Eq. (1.5) takes the form (see, for example, [12])

$$\frac{\partial \langle n_c \rangle}{\partial t} = 2\mu\Delta\langle n_c\rangle + 2\frac{\partial^2}{\partial z_i \partial z_j}[B_{ij}(\mathbf{z})\langle n_c\rangle], \quad B_{ij}(\mathbf{z}) = \mathcal{D}_{ij}(0) - \mathcal{D}_{ij}(\mathbf{z}) \tag{1.10}$$

In addition to the turbulent fluctuations of the velocity field of the medium, this equation takes into account the molecular diffusion of particles with molecular diffusivity μ, which plays a principal role when analysing the evolution of the impurity density (see, for example, the monograph [13], where the role of molecular diffusion when describing impurity density fluctuations is discussed in detail).

For a radially symmetric distribution $g = g(z, t)$, Eq. (1.10) becomes

$$\frac{\partial \langle n_c \rangle}{\partial t} = \frac{2}{z^{d-1}}\frac{\partial}{\partial z}z^{d-1}\left[\frac{\partial}{\partial z}[(\mu + P_{\|} + E_{\|})\langle n_c\rangle] + Q\langle n_c\rangle\right] \tag{1.11}$$

Here

$$P_{\|}(z) = \frac{d^2A_p(z)}{dz^2} + \mathcal{D}_p, \quad E_{\|}(z) = (d-1)\left[\frac{1}{z}\frac{dA_e(z)}{dz} + \mathcal{D}_e\right]$$
$$Q(z) = (d-1)\frac{d}{dz}\left(\frac{1}{z}\frac{dA_p(z)}{dz}\right) - \frac{dE_{\|}(z)}{dz} \tag{1.12}$$

d is the dimensionality of the space, and the following asymptotic expansion is used

$$A_{e,p}(z) = A_{e,p}(0) - \frac{z^2}{2}\mathcal{D}_{e,p} + \frac{z^4}{24}B_{e,p} - \ldots \quad (z \to 0) \tag{1.13}$$

which specifies a nature of the coefficients $\mathcal{D}_{e,p}$ in relations (1.12). It is also assumed that the functions $A_{e,p}(z)$ satisfy the condition of attenuation of the correlations with distance and tend to zero quite rapidly as $z \to \infty$, which ensures convergence of the integrals which arise below.

The steady density $n_{st}(z)$ (1.6) is subject to the boundary-value problem (1.7)

$$\frac{d}{dz}[(\mu + P_{\|} + E_{\|})n_{st}] + Qn_{st} = 0, \quad n'_{st}(0) = 0, \quad n_{st}(\infty) = n_0$$

and has the following solution

$$n_{st}(z) = n_0 \exp\left[\int_z^{\infty}\frac{dP(y)}{\mu + P_{\|}(y) + E_{\|}(y)}\right] \tag{1.14}$$

Here

$$P(z) = \Delta A_p(z) + d\mathcal{D}_p = \frac{1}{z^{d-1}}\frac{d}{dz}\left(z^{d-1}\frac{dA_p(z)}{dz}\right) + d\mathcal{D}_p \tag{1.15}$$

We will discuss the physical meaning of the functions in expression (1.14) by splitting the field of the particle velocities into divergent and vortical parts

$$\boldsymbol{v}(\mathbf{x}, t) = \boldsymbol{v}^p(\mathbf{x}, t) + \boldsymbol{v}^e(\mathbf{x}, t)$$

Suppose the parallel correlation functions of the velocities are known

$$B_{\parallel}^{e,p}(z,\theta) = \langle(\mathbf{v}^{e,p}(\mathbf{x},t)\cdot\mathbf{l})(\mathbf{v}^{e,p}(\mathbf{x}+\mathbf{z},t+\theta)\cdot\mathbf{l})\rangle, \quad \mathbf{l} = \mathbf{z}/z$$

They specify coefficients of turbulent diffusion

$$D_{\parallel}^{p}(z) = 2\int_0^{\infty} B_{\parallel}^{p}(z,\theta)d\theta, \quad D_{\parallel}^{e}(z) = 2\int_0^{\infty} B_{\parallel}^{e}(z,\theta)d\theta$$

The corresponding coefficients of relative diffusion occur in the denominator of the integral in equality (1.14):

$$P_{\parallel}(z) = \mathfrak{D}_p - D_{\parallel}^{p}(z), \quad E_{\parallel}(z) = (d-1)\mathfrak{D}_e - D_{\parallel}^{e}(z)$$

The function $P(z)$ in the numerator of this integral has a similar meaning. We will show this by a determining the mean of the scalar product

$$B^{p}(z,\tau) = \langle \mathbf{v}^{p}(\mathbf{x},t)\cdot\mathbf{v}^{p}(\mathbf{x}+\mathbf{z},t+\tau)\rangle$$

The corresponding coefficient of relative diffusion is

$$P(z) = d\mathfrak{D}_p - D^{p}(z), \quad D_p(z) = 2\int_0^{\infty} B^{p}(z,\tau)d\tau$$

This coefficient is connected with the coefficient of parallel relative diffusion of the divergent part of the velocity field by Obukhov's relation

$$P(z) = P_{\parallel}(z) + \frac{d-1}{z}\int_0^{z} P_{\parallel}(y)dy \tag{1.16}$$

Thus, from relation (1.14) it can be seen that the greater the total energy of the divergent part of velocity field the stronger the localization, and the greater the parallel coefficients of relative diffusion of the divergent and vortical parts, the weaker the localization.

We will investigate the form $n_{\text{st}}(z)$ by writing expression (1.14) as

$$n_{\text{st}}(z) = N\exp\left[-\int_0^{z}\frac{dP(y)}{\mu + P_{\parallel}(y) + E_{\parallel}(y)}\right], \quad N = n_{\text{st}}(0) \tag{1.17}$$

Suppose l_υ is the internal scale of turbulence of the field $\mathbf{v}(\mathbf{x},t)$. If $z \ll l_\upsilon$ the following asymptotic forms hold

$$E_{\parallel}(z) \simeq \frac{d-1}{6}B_e z^2, \quad P_{\parallel}(z) \simeq \frac{1}{2}B_p z^2, \quad P(z) \simeq \frac{d+2}{6}B_p z^2 \tag{1.18}$$

Here we have used expansions (1.13) and relations (1.12) and (1.16). Substituting expressions (1.18) into equality (1.17) we obtain

$$n_{\text{st}}(z) = N\left[\frac{l_n^2}{l_n^2 + (3\gamma + d - 1)z^2}\right]^{\chi} \tag{1.19}$$

where

$$l_n^2 = \frac{6\mu}{B_e}, \quad \gamma = \frac{B_p}{B_e}, \quad \chi = \frac{(d+2)\gamma}{3\gamma + d - 1} \tag{1.20}$$

We will investigate the effect of competition between the divergent and vortical components of the velocity field on the steady density (1.14). To do this we will introduce the following dimensionless functions and dimensionless parameters

$$e_{\|}(z) = \frac{E_{\|}(z)}{\mathfrak{D}_e}, \quad p(z) = \frac{P(z)}{\mathfrak{D}_p}, \quad p_{\|}(z) = \frac{P_{\|}(z)}{\mathfrak{D}_p}, \quad \delta = \frac{\mathfrak{D}_p}{\mathfrak{D}_e}, \quad \nu = \frac{\mu}{\mathfrak{D}_e}$$

and rewrite expression (1.14) as follows:

$$n_{st}(z) = n_0 \exp\left[\delta\int_z^{\infty}\frac{dp(y)}{\nu + \delta p_{\|}(y) + e_{\|}(y)}\right]$$

Hence it can be seen that when $\delta \ll 1$ the steady density is in fact equal to some δ-independent function to the power δ

$$n_{st}(z) \simeq n_0\rho^{\delta}(z), \quad \rho(z) = \exp\left[\int_z^{\infty}\frac{dp(y)}{\nu + e_{\|}(y)}\right] \tag{1.21}$$

In particular, the maximum steady density around the tagged particle is equal to

$$N \simeq n_0\rho^{\delta}(0)$$

We will estimate the quantity $\rho(0)$ assuming $e_{\|}(z) \simeq p(z)$. As a result, from expression (1.21) for $\rho(z)$ we obtain the estimate $\rho(0) \simeq \mathrm{Pe} = \mathfrak{D}_e/\mu$, that is,

$$N \simeq n_0(\mathrm{Pe})^{\delta}$$

Here Pe is the Peclet number of the vortical component of the impurity velocity.

If, for example, $\mathrm{Pe} \sim 10^{10}$ and $\delta \sim 10^{-1}$, then $N \sim 10n_0$, that is, for these parameters of the turbulence the density inside the cluster is only one order of magnitude greater than the mean density.

2. A PROBABILISTIC INTERPRETATION

We recall that not only the mean density around the tagged particle, but also the distribution of the distances between the particles $g(z, t)$ satisfies Eq. (1.11). The normalized solution of Eq. (1.11), i.e. which satisfies the equality

$$\int g(z; t)d\mathbf{z} = 1$$

specifies the distribution of the distances between particles. Unlike the mean density, the density $g(z, t) \to 0$ as $t \to \infty$, since the particles, finally, fall into different clusters and diffuse independently. Nevertheless, when the Peclet number is large, particles form quasistable pairs, which govern the localization. The specific features of the behaviour of such pairs of particles can be studied under the assumption that for any z the asymptotic forms (1.18) hold.

We find the steady distribution of the distances between the particles by rewriting expression (1.19) in a somewhat different from

$$g_{st}(y) = \frac{C}{[1 + (3\gamma + d - 1)y^2]^{\chi}}, \quad y = \frac{z}{l_n} \tag{2.1}$$

Here C is a normalizing constant. If $C > 0$, a steady distribution exists. Physically this means that the "attraction" between particles in the cluster is so great that they are not dispersed. If $C = 0$, the particles may be dispersed to as large a distance as desired.

The case $C > 0$ will be called superlocalization. From relation (2.1) it can be seen that its existence depends on the exponent χ. The latter is specified by the parameter γ, which expressed the relative contribution of the divergent part of the particle velocity. It can be shown that $C > 0$ if $2\chi > d$, that is, if

$$\gamma > d(d-1)/(4-d)$$

The analysis of distribution (2.1) of the distance between particles enables one to judge the characteristic form of the clusters and its variation as the parameter γ increases. We will discuss in detail the two-dimensional case ($d = 2$), which occurs for a floating impurity. Then the quantity $g_{st}(y)$ in Eq. (2.1) is a two-dimensional distribution of the coordinates $\{y_1, y_2\}$ of the dimensionless vector $\mathbf{y}$, which joins two particles.

Note that distribution (2.1) of the components of the vector $\mathbf{y}$ does not decompose into the product of distributions only for y_1 and only for y_2. Hence, the components of the vector $\mathbf{y}$ are statistically dependent.

We will examine some features of the dependence mentioned above. We write the unconditional distribution of the component y_1

$$g_1(y) = \int_{-\infty}^{\infty} g_{st}(\sqrt{y^2 + y_2^2})dy_2$$

Substituting $g_{st}(y)$ (2.1) into this equation and evaluating the integral we obtain

$$g_1(y) = f\left(\frac{5\gamma - 1}{6\gamma + 2}, y\right); \quad f(\chi, y) = \frac{\sqrt{3\gamma + 1}\,\Gamma(\chi)}{\sqrt{\pi}\,\Gamma(\chi - 1/2)[1 + (3\gamma + 1)y^2]^{\chi}} \tag{2.2}$$

The statistical dependence of the components of the vector $\mathbf{y}$ means that the distribution of the component y_1 is not equal to its conditional distribution $g_1(y|y_2)$ obtained under the condition that y_2 has a specified value. Calculations for the case $y_2 = 0$ give

$$g_1(y|0) = f\left(\frac{4\gamma}{3\gamma + 1}, y\right) \tag{2.3}$$

Note that the conditional distribution (2.3) is also positive when the unconditional distribution (2.2) vanishes identically: if the distribution $g_1(y) > 0$ only when $\gamma > 1$, then $g_1(y|0) > 0$ while $\gamma > 1/5$. The latter is explained by the strong anisotropy of the clusters of particles, which arises owing to the fact that there are preferred directions along which the particles are "attracted" more strongly than in other directions.

We will demonstrate this by the simplest model. Let us take the unit vector

$$\mathbf{m} = \mathbf{i}_1 \cos\alpha + \mathbf{i}_2 \sin\alpha$$

where $\mathbf{i}_1$ and $\mathbf{i}_2$ are the basis vectors of a Cartesian system of coordinates and α is the angle that specifies the orientation of the vector $\mathbf{m}$. We will construct the conditional distribution $g(\mathbf{y}|\alpha)$ of the components of the vectory $\mathbf{y}$ under the condition that the predominant direction of localization is perpendicular to the vector $\mathbf{m}$:

$$g(\mathbf{y}|\alpha) = \phi(\mathbf{m}\cdot\mathbf{y})\psi(y) \tag{2.4}$$

The first factor takes into account the localization in a direction normal to the vector $\mathbf{m}$, and the second factor accounts for the isotropic "remaining" localization in all directions. We will choose the factors so that the mean of conditional distribution (2.4) over the angle α gives distribution (2.1)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} g(\mathbf{y}|\alpha)d\alpha = g_{st}(y) \tag{2.5}$$

This is actually so if

$$\phi(\mathbf{m}\cdot\mathbf{y}) \sim \frac{1}{1 + \kappa(\mathbf{m}\cdot\mathbf{y})^2}, \quad \psi(y) \sim \frac{1}{(1 + \kappa y^2)^{\rho}} \tag{2.6}$$

Let us average equality (2.5) over the angle α. Noting that

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{d\alpha}{1 + \kappa(\mathbf{m}\cdot\mathbf{y})^2} = \frac{1}{\sqrt{1 + \kappa y^2}}$$

we return to steady distribution (2.1), where

$$d = 2, \quad \chi = \rho + 1/2 \quad (\kappa = 3\gamma + 1)$$

Therefore, we have the following transformation of the form of the cluster as the parameter γ changes. When

$$\rho > 1/2 \Rightarrow \chi > 1 \Rightarrow \gamma > 1$$

the radially symmetric factor (2.6) predominates in conditional distribution (2.4), whereas the cluster is approximately isotropic with characteristic size l_n. Such clusters will be called *circles*. This case corresponds to superlocalization and reveals the geometrical essence of this concept: under superlocalization conditions the cluster sizes are of the order of $l_n \sim \sqrt{\mu/B_p}$, and the clusters themselves are in fact isotropic. If

$$0 < \rho < 1 \Rightarrow 1/2 < \chi < 1 \Rightarrow 1/5 < \gamma < 1$$

the clusters have a prolate form. We shall call these *filaments*. As before, the thickness of the filaments is of the order of l_n while the length is determined by the external turbulence scale L_υ. When

$$\rho < 0 \Rightarrow 0 < \chi < 1/2 \Rightarrow 0 < \gamma < 1/5$$

the effective sizes of the clusters in all directions are of the order L_υ. These clusters will be called *protoclusters*.

Hence, when crossing the thresholds $\gamma = 1/5$ and $\gamma = 1$ there are peculiar kinds of phase transitions: the form of the cluster changes from protoclusters to filaments, and then to k circles.

In the three-dimensional case we shall restrict our consideration to listing the regions of γ axis, which correspond to different forms of clusters, obtained from an analysis of conditional distributions. This is the regime of superlocalization

$$3/2 < \chi \Rightarrow \gamma > 6$$

which corresponds to compact nearly isotropic clusters, and also the regimes

$$1 < \chi < 3/2 \Rightarrow 1 < \gamma < 6$$

$$1/2 < \chi < 1 \Rightarrow 2/7 < \gamma < 1$$

$$\chi < 1/2 \Rightarrow \gamma < 2/7$$

3. LOCALIZATION OF A FLOATING IMPURITY

Up to now the ratio of the divergent and vortical parts of the particle velocity was assumed to be arbitrary. However, for particles floating on the surface of an incompressible fluid the quantity γ takes a well-defined value. We shall show this by discussing the simplest model of a floating impurity. Suppose the impurity moves along a plane inside a turbulent medium and the velocity of the impurity particles is equal to a projection of the statistically uniform three-dimensional turbulent velocity field onto this plane. We will obtain the steady density of such a "floating" impurity by noting that the two-dimensional potentials of the divergent and vortical parts of the impurity velocity are related to the vortical potential of three-dimensional motion by the equalities

$$\Delta^2 A_e^2(z) = \Delta^3 A_e^3(z), \quad A_e^2(z) - A_p^2(z) = A_e^3(z) \tag{3.1}$$

The potential $A^d(z)$ describes the correlation properties of the d-dimensional velocity field and Δ^d is the d-dimensional Laplacian.

We will find functions that specify the steady solution

$$n_{\mathrm{st}}(z) = n_0 \exp\left[\int_z^\infty \frac{dP^2(y)}{\mu + P_\parallel^2(y) + E_\parallel^2(y)}\right] \tag{3.2}$$

From relations (1.15) and (1.3) we have

$$P^2(z) = (\Delta^3 - \Delta^2)A_e^3(z) + \mathfrak{D}_e^3 = \frac{1}{2}E_{\|}^3(z) \tag{3.3}$$

Here formulae (1.12) have been taken into account. Hence, according to expression (3.3), the function $P^2(z)$, which is responsible for the localization of floating impurity, is equal to half the longitudinal coefficient of relative diffusion $E_{\|}^3(z)$ of the three-dimensional vortical velocity field. Let us find what the denominator of the integrand in formula (3.2) is equal to. By relations (1.12) and (3.1) we have

$$P_{\|}^2(z) + E_{\|}^2(z) = E_{\|}^3(z) \tag{3.4}$$

Substituting expressions (3.3) and (3.4) into equality (3.2) and changing to dimensionless parameters, we find

$$g_{\mathrm{st}}(z) = \sqrt{\frac{1+\mathrm{Pe}}{1+\mathrm{Pe}\,e(z)}}, \quad \mathrm{Pe} = \frac{2\mathfrak{D}_e^3}{\mu}, \quad e(z) = \frac{E_{\|}^3(z)}{2\mathfrak{D}_e^3} \tag{3.5}$$

Expression (3.5) enables us to study in detail the properties of localization of a floating impurity. We will do this in the model case of a Gaussian correlation function

$$e(z) = 1 - \exp\left(-\frac{z^2}{2l_\upsilon^2}\right)$$

Substituting this expression into formula (3.5) and changing to the dimensionless coordinate $y = z/l_\upsilon$, we obtain

$$g_{\mathrm{st}}(y) = \frac{1}{\sqrt{1-\eta e^{-y^2/2}}}, \quad \eta = \frac{\mathrm{Pe}}{1+\mathrm{Pe}}$$

We will calculate a mean mass of the cluster defined by formulae (2.5)

$$M = n_0 \iint [g_{\mathrm{st}}(z) - 1]\,d^2z$$

Using the radial symmetry of the mean density, we will have

$$M = \mathcal{M}\int_0^\infty \left[\frac{1}{\sqrt{1-\eta e^{-x}}} - 1\right]dx = \mathcal{M}m(\mathrm{Pe})$$

$$\mathcal{M} = n_0 \pi l_\upsilon^2, \quad m(\mathrm{Pe}) = \ln\left(4 + \frac{4}{\mathrm{Pe}}\right) - 2\,\mathrm{arctg}\sqrt{\frac{1}{1+\mathrm{Pe}}}$$

Note that, whereas the maximum mean density around the tagged particle increases as a square root of the Peclet number, the mean mass of the cluster tends to a limit that is independent of the Peclet number:

$$m(\mathrm{Pe}) = \ln 4 - \frac{2}{\sqrt{\mathrm{Pe}}} + O\left(\frac{1}{\mathrm{Pe}}\right) \quad (\mathrm{Pe} \to \infty)$$

In addition, we note that solution (3.5) corresponds to the boundary case $\gamma = 1/5$. The latter means that the characteristic form of clusters of floating impurity is some mixture of fibres and protoclusters.

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REFERENCES

1. ELPERIN, T., KLEEORIN, N. and ROGACHEVSKII, I., Dynamics of the passive scalar in compressible turbulent flow: Large-scale patterns and small-scale fluctuations. *Phys. Rev. E.*, 1995, **52**, 3, 2617–2634.
2. OTT, S. and MANN, J., An experimental investigation of the relative diffusion of particle pairs in three-dimensional turbulent flow. *J. Fluid. Mech.*, 2000, **422**, 207–223.
3. ECKHARDT, B. and SCHUMACHER, J., Turbulence and passive scalar transport in a free-slip surface. *Phys. Rev. E,* 2001, **64**, 1, Paper 016314.
4. BALKOVSKY, E., FALKOVICH, G. and FOUXON, A. Intermittent distribution of inertial particles in turbulent flows. *Phys. Rev. Lett.*, 2001, **86**, 13, 2790–2793.
5. SAICHEV, A. I., and ZHUKOVA, I. S., The arising and evolution of the passive tracer clusters in compressible random media. *Lecture Notes in Physics*, 1998, 511, 353–371.
6. ZHUKOVA, I. S. and SAICHEV, A. I., The localization of clusters of floating particles on the surface of turbulent flow. *Prikl. Mat. Mekh.* 2004, **64**, 2, 624–630.
7. MAXEY, M. R., The gravitational settling of aerosol particles in homogeneous turbulence and random flow fields. *J. Fluid Mech.*, 1987, 174, 441–465.
8. KRAICHNAN, R. H., Diffusion by a random velocity field. *Phys. Fluids*, 1970 **13**, 1, 22–31.
9. GURBATOV, S. N., MALAKHOV, A. N. and SAICHEV, A. I., *Nonlinear Random Waves and Turbulence in Nondispersive Media: Waves, Rays, Particles.* Manchester University Press, Manchester, 1991.
10. SAICHEV, A. I. and WOYCZYNSKI, W. A., Probability distributions of passive tracers in randomly moving media. In *Stochastic Models in Geosystems* (Edited by Molchanov S. A. and Woyczynski W. A.): Springer, New York, 1997, 359–399.
11. KLAYTSKIN, V. I. and SAICHEV, A. I., The statistical theory of a floating impurity in a random velocity field. *Zh. Eksp. Teor. Fiz.* 1997, **111**, 4, 1297–1313.
12. KLAYTSKIN, V. I., *The Stochastic Equations from the Point of View of a Physicist.* Nauka, Moscow, 2001.
13. KLYATSKIN, V. I., *The Dynamics of Stochastic Systems.* Fizmatgiz, Moscow, 2002.

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